

$$P = \frac{F}{A}$$

Module 1:-

Simple stresses & strain.

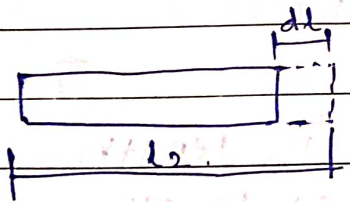
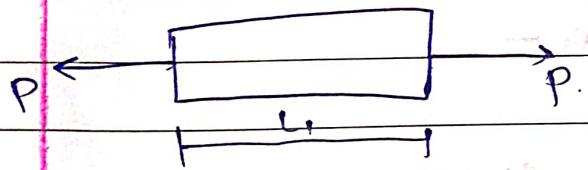
stress (σ) - stress is defined as the resistance offered by the body to the deformation

Mathematically,

$$\text{stress } (\sigma) = \frac{\text{resisting force } (P)}{\text{Area}}$$

units = N/m^2 (pascal).

strain (e) - It is defined as the ratio of change in dimensions (length, width, thickness) to the original dimension (l, w, t)

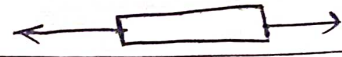


$$\text{strain } (e) = \frac{l_2 - l_1}{l_1}$$

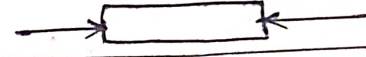
$$e = \frac{dl}{l_1}$$

strain has no units.

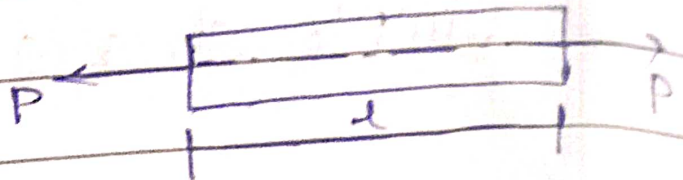
tensile force - pulling



compressive - pushing



1. Tensile stress (σ_t) -



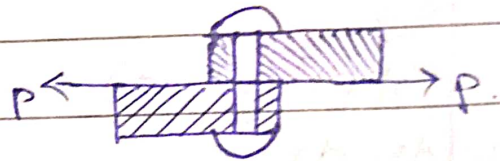
$$\sigma_t = \frac{\text{Pulling force (or) tensile force}}{\text{Area}}$$

2. compressive stress (σ_c) -



$$\sigma_c = \frac{\text{Pushing force (or) compressive force}}{\text{Area}}$$

3. shear stress (σ_s) -



$$\sigma_s = \frac{\text{Shear force (or) tangential force}}{\text{Area}}$$

* Types of strain -

1. Tensile strain (e_t) -

$$e_t = \frac{\text{Increase in length}}{\text{Original length}}$$

2. compressive strain (e_c) -

$$e_c = \frac{\text{decrease in length}}{\text{Original length}}$$

3. Shear strain (e_s) - relationship betⁿ stress & strain (or) hook's law.

It states that within proportionality limit stress is directly proportional to strain.

i.e. stress \propto strain

$$\sigma \propto e$$

$$\boxed{\sigma = Ee} \quad \text{--- (1)}$$

where,

E is young's modulus. (or) modulus of elasticity.

Eqⁿ (1) can be rewritten as,

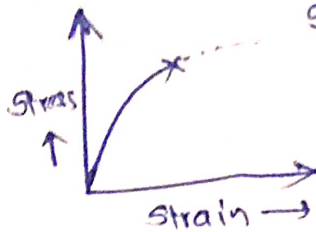
$$\sigma = Ee$$

$$\frac{P}{A} = E \cdot \frac{dl}{l}$$

$$\boxed{dl = \frac{P \cdot l}{A \cdot E}}$$

extension (or)
change in length

Upper yield point is the point of starting permanent deformation & due to permanent deformation strain will increase & stress will decrease.

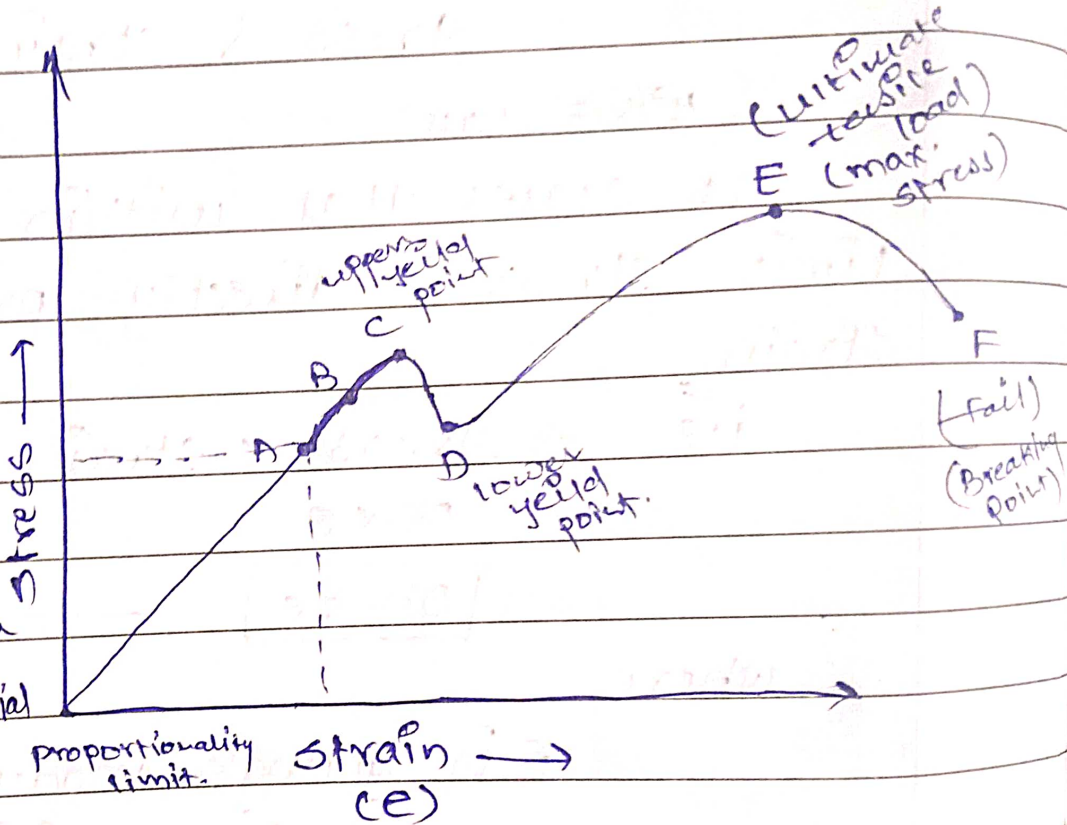


Stress strain dim for brittle material.
Date: / / 20

Draft

Stress strain dim for Ductile material.

Additional stress is req. to overcome the carbon's resistance, at which point everything can move uninterrupted, which means that the material can then continue to yield @ lower stress



Q. Modulus of rigidity or shear modulus -
(C, N, G)

It is a ratio of shear stress to the shear strain.

$$C = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{e_s}$$

* Factor of safety -

It is a ratio of ^{maximum} ultimate stress to the permissible stress.

$$FOS = \frac{\text{ultimate stress}}{\text{permissible stress}}$$

~~Q.~~

A tensile test was conducted on a mild steel bar. the following data was obtained from the test.

i) Dia of steel bar = 3 cm

ii) Gauge length of bar = 20 cm.

iii) load @ elastic limit = 250 kN

iv) Extension @ a load of 150 kN = 0.21 mm.

v) max load = 380 kN.

vi) Total extension = 60 mm.

vii) Dia of rod @ failure = 2.25 cm

determine

i) young's modulus

ii) stress @ elastic limit

iii) % elongation

iv) % decrease in area.

$$\rightarrow \text{dia of rod} = 3 \text{ cm} \\ = 0.03 \text{ m}$$

$$\text{Area} = \frac{\pi (0.03)^2}{4} = 7.06 \times 10^{-4} \text{ m}^2$$

$$\text{length of bar} = 20 \text{ cm} \\ = 0.2 \text{ m}$$

$$1) E = \sigma \times e$$

$$\sigma = Ee$$

$$E = \frac{\sigma}{e}$$

$$\sigma = \frac{F}{A} = \frac{150 \times 10^3}{7.06 \times 10^{-4}} = 212464589.2 \frac{\times 10^6}{10^6}$$

$$= 212.4645 \text{ N/mm}^2$$

$$e = \frac{\text{extension}}{\text{original}} = \frac{0.21 \text{ mm}}{200 \text{ mm} (2 \text{ cm})}$$

$$e = 1.05 \times 10^{-3} = 0.00105$$

$$E = \frac{212.4645 \times 10^6}{0.00105} = 2.03 \times 10^{11} \text{ N/m}^2$$

$$2) \text{ stress @ elastic limit} = \frac{\text{load}}{\text{Area}}$$

$$= \frac{250 \times 10^3}{7.06 \times 10^{-4}}$$

$$= 354.10 \times 10^6 \text{ N/m}^2$$

(Extension)

$$3\% \text{ elongation} = \frac{\text{Total increase in length}}{\text{Original length}} \times 100$$

$$= \frac{60 \text{ mm}}{200 \text{ mm (20 cm)}} \times 100$$

$$= \underline{\underline{30\%}}$$

Failure

$$4\% \text{ decrease in area} = \frac{\text{Original Area} - \text{Area @}}{\text{Original Area}}$$

$$\text{Dia of rod @ failure} = 2.25 \text{ cm} = \frac{2.25}{100} = 0.0225 \text{ m}$$

$$\text{Area of rod @ failure} = \frac{\pi (0.0225)^2}{4} = 3.976 \times 10^{-4} \text{ m}^2$$

$$\therefore = \frac{7.06 \times 10^{-4} - 3.976 \times 10^{-4}}{7.06 \times 10^{-4}} \times 100$$

$$= \underline{\underline{43.76\%}}$$

Increased

Q. The ultimate stress, for a hollow steel column which carries an axial load of 1.9 mega N. is 480 N/mm^2 . If the external dim of column is 200 mm., Determine internal dim take EoS as 4.

→ Given - external dim $D = 200 \text{ mm}$.

ultimate stress = 480 N/mm^2 .

Let 'd' is internal dim.

$$FOS = 4;$$

$$FOS = \frac{\text{ultimate stress}}{\text{working stress}}$$

$$W.S = \frac{\text{ultimate}}{FOS} = \frac{480}{4} = 120 \text{ N/mm}^2$$

$$\text{Working stress} = \frac{\text{load}}{\text{Area}}$$

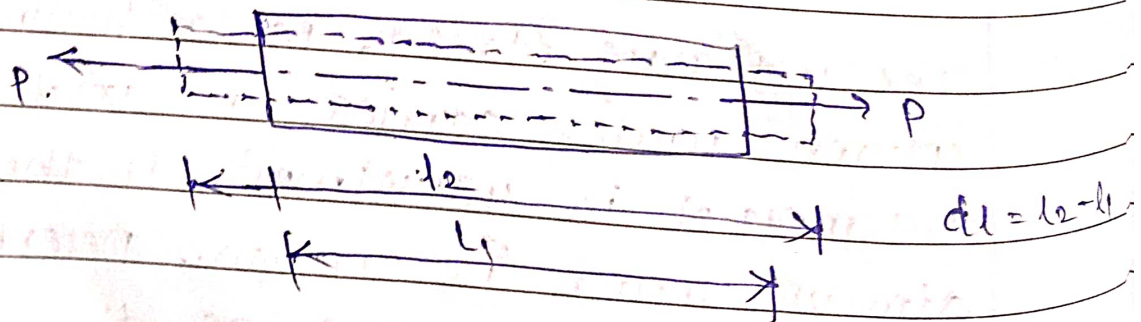
$$120 = \frac{1.9 \times 10^6}{\frac{\pi}{4}(200^2 - d^2)}$$

$$d = 140.85 \text{ mm.}$$

20/08/2021

* Poisson's Ratio -
(μ or ν).

It is defined as the ratio of lateral strain to the longitudinal (linear) strain.



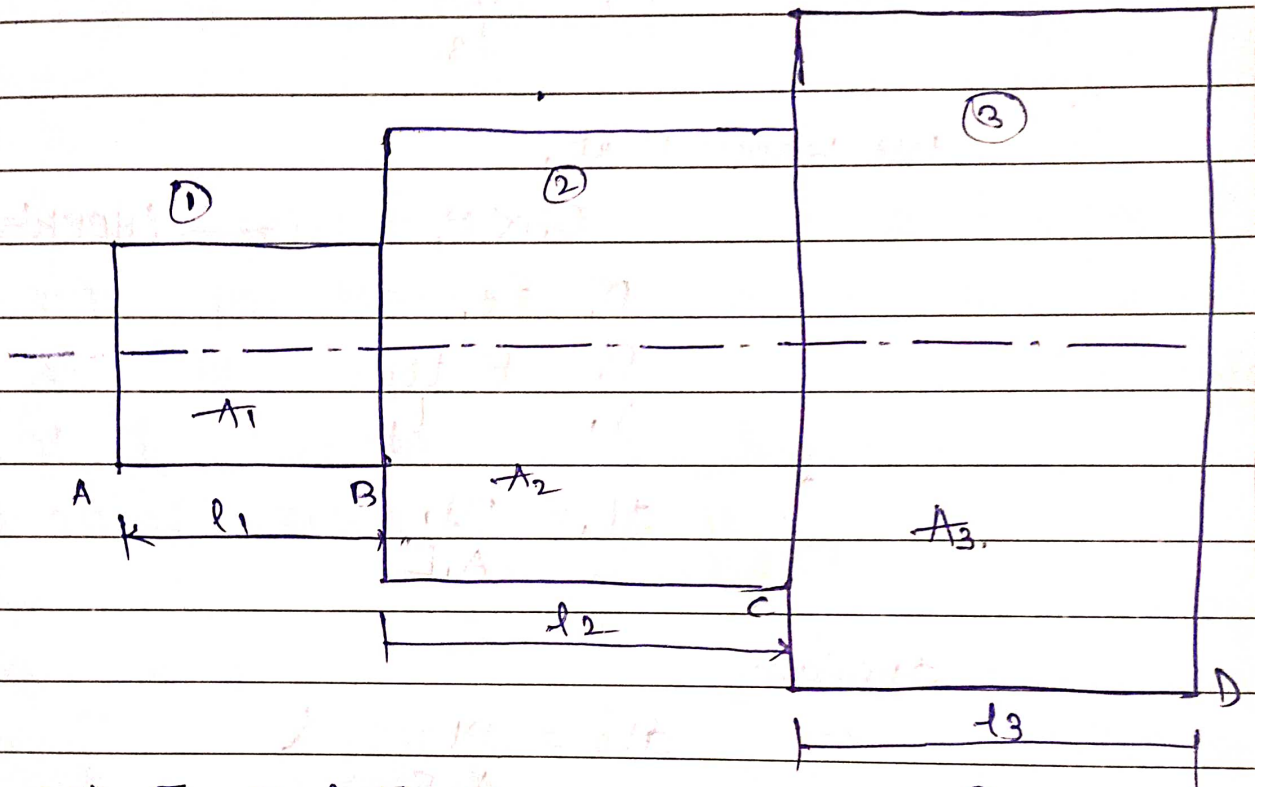
i.e $\mu = \frac{\text{lateral strain}}{\text{longitudinal (or) linear strain}}$

$$\mu = - \frac{e_2}{e_1}$$

$$\therefore e_2 = -\mu e_1$$

(-ve sign indicates decrease in lateral dimensions).

* Analysis of bars of varying sections.



Let σ_1, σ_2 & σ_3 be the stresses induced in the parts AB, BC, CD

similarly, let e_1, e_2 & e_3 be the strains induced in parts AB, BC & CD

stress (σ_1) in the ^{part} ~~along~~ AB = $\frac{P}{A_1}$

similarly, ^(BC) $\sigma_2 = \frac{P}{A_2}$ &

(CD) $\sigma_3 = \frac{P}{A_3}$

$$e_1 = \frac{dl_1}{l_1}$$

similarly,

$$e_2 = \frac{dl_2}{l_2}$$

$$e_3 = \frac{dl_3}{l_3}$$

We know that,

$$\sigma_1 \propto e_1$$

— (HOOK'S LAW).

$$\sigma_1 = E e_1$$

$$\frac{P_1}{A_1} = E \frac{dl_1}{l_1}$$

$$dl_1 = \frac{P l_1}{A_1 E}$$

similarly,

$$dl_2 = \frac{P l_2}{A_2 E}$$

$$dl_3 = \frac{P l_3}{A_3 E}$$

The total extension (or) increase in length of bar is $dl = dl_1 + dl_2 + dl_3$

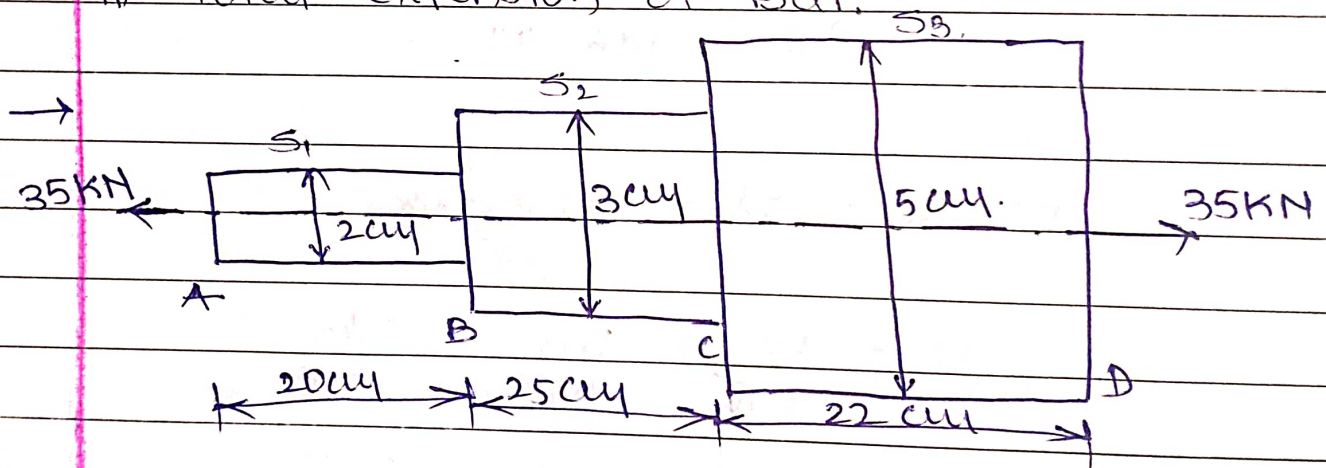
$$\therefore dl = \frac{P l_1}{A_1 E} + \frac{P l_2}{A_2 E} + \frac{P l_3}{A_3 E}$$

$$dl = \frac{P}{E} \left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} \right)$$

If the young's modulus of diff section parts is different, then the total change in the length of the bar, is

$$\text{i.e. } \Delta l = P \left(\frac{l_1}{A_1 E_1} + \frac{l_2}{A_2 E_2} + \frac{l_3}{A_3 E_3} \right)$$

- Q. An axial pull of 35000 N is acting on a bar consisting of 3 lengths as shown in fig. If young's modulus is $2 \times 10^5 \text{ N/mm}^2$ find i) stresses in each section
ii) total extension of bar.



Given - $E = 2 \times 10^5 \text{ N/mm}^2$.

$$d_1 = 2 \text{ cm} = 20 \text{ mm}$$

$$A_1 = \frac{\pi (d_1)^2}{4} = \frac{\pi (20)^2}{4} = 314.15 \text{ mm}^2$$

$$l_1 = 20 \text{ cm} = 200 \text{ mm}$$

$$\sigma_1 = \frac{P}{A_1} = \frac{35000}{314.15} = 111.41 \text{ N/mm}^2$$

$$d_2 = 3 \text{ cm} = 30 \text{ mm}$$

$$A_2 = \frac{\pi (30)^2}{4} = 706.85 \text{ mm}^2$$

$$L_2 = 25 \text{ cm} = 250 \text{ mm}$$

$$\sigma_2 = \frac{P}{A_2} = \frac{35000}{706.85} = 49.51 \text{ N/mm}^2$$

$$d_3 = 5 \text{ cm} = 50 \text{ mm}$$

$$A_3 = \frac{\pi (50)^2}{4} = 1963.49 \text{ mm}^2$$

$$L_3 = 22 \text{ cm} = 220 \text{ mm}$$

$$\sigma_3 = \frac{P}{A_3} = \frac{35000}{1963.49} = 17.82 \text{ N/mm}^2$$

iii) $\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$

$$\Delta l = \frac{Pl_1}{A_1E} + \frac{Pl_2}{A_2E} + \frac{Pl_3}{A_3E}$$

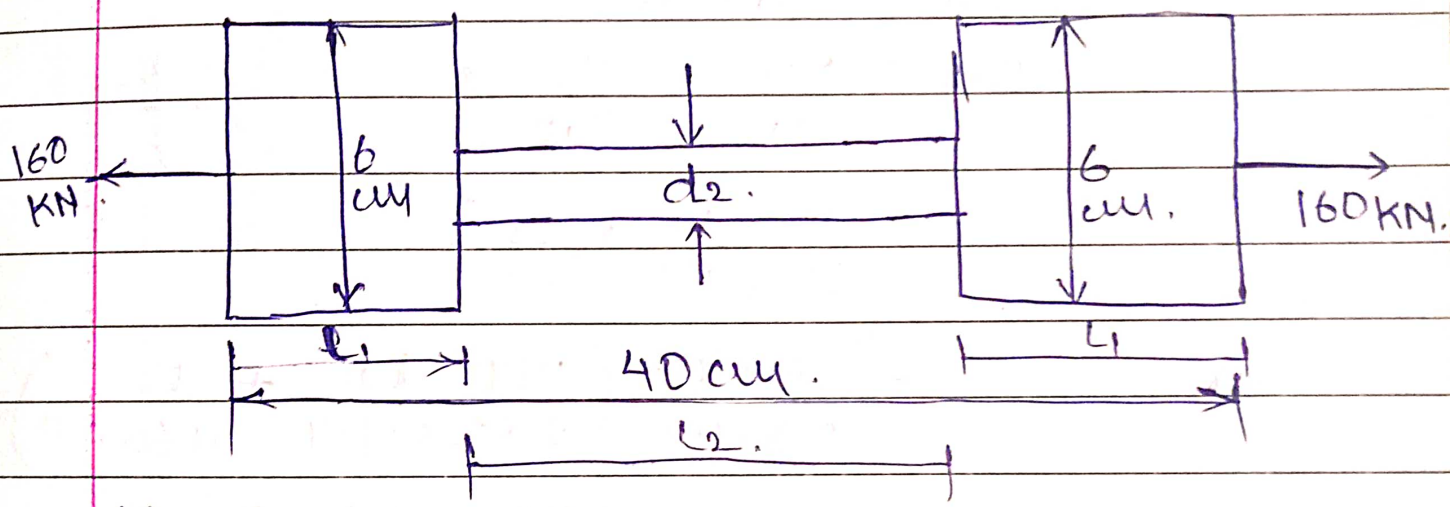
$$= \frac{P}{E} \left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} \right]$$

$$= \frac{35000}{2 \times 10^5} \left[\frac{200}{314.15} + \frac{250}{706.8} + \frac{220}{1963.49} \right]$$

$\Delta l = 0.19 \text{ mm}$

Q.1

A bar shown in fig is subjected to a tensile load of 160 kN. If the stress in middle portion is limited to 150 N/mm² determine the diameter of middle portion. Find also the length of middle portion. If total elongation of bar is to be 0.2 mm. Take $E = 2 \times 10^5 \text{ N/mm}^2$.



Given -

$$E = 2 \times 10^5 \text{ N/mm}^2.$$

$$\Delta L = 0.2 \text{ mm}.$$

$$P = 160 \times 10^3 \text{ N}.$$

$$\sigma_2 = 150 \text{ N/mm}^2.$$

$$\sigma_2 = \frac{P}{A_2}.$$

$$150 = \frac{160 \times 10^3}{\pi/4 (d_2)^2}.$$

$$d_1 = 6 \text{ cm} = 60 \text{ mm}.$$

$$A_1 = \frac{\pi (60)^2}{4} = 2827.4 \text{ mm}^2.$$

$$d_2 = \underline{\underline{36.85 \text{ mm}}}.$$

Let L_2 be the length of the middle portion. & L_1 be the length of end portions.

From the figure

$$2L_1 + L_2 = 400 \text{ mm.}$$

$$L_1 = \frac{400 - L_2}{2}$$

$$\Delta l = \Delta l_1 + \Delta l_2$$

$$0.2 = \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E}$$

$$= \frac{P}{E} \left(\frac{L_1}{A_1} + \frac{L_2}{A_2} \right)$$

$$= \frac{160 \times 10^3}{2 \times 10^5} \left(\frac{400 - L_2}{2 \times 2827.4} + \frac{L_2}{1066.50} \right)$$

$$\therefore L_2 = \underline{\underline{236.57 \text{ mm}}}$$

* Principle of Superposition

It states that resultant strain is equal to the algebraic sum of the strain caused by the individual loads (forces).

~~Q.1~~
Q.

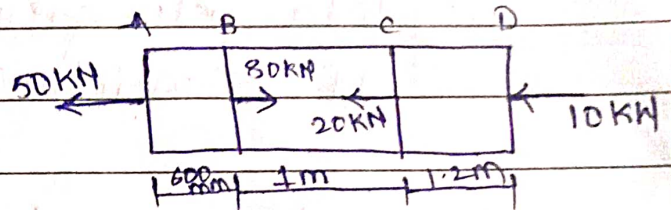
$dL = ?$

A brass bar having cross sectional area of 1000 mm^2 subjected to axial force as shown in fig. Find the total elongation of bar $E = 1 \times 10^5 \text{ N/mm}^2$.

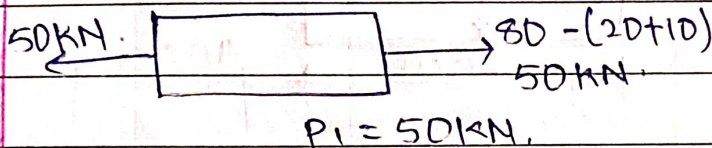
→ Given -

$$E = 1 \times 10^5 \text{ N/mm}^2$$

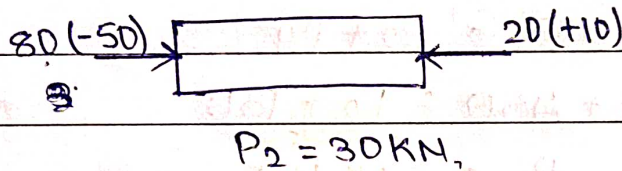
$$A = 1000 \text{ mm}^2$$



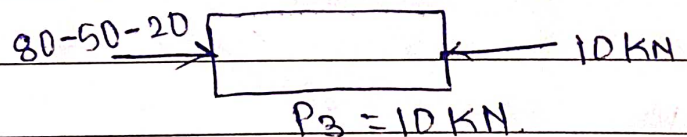
FBD part AB



Part BC



Part CD



$$dL = dL_1 - dL_2 - dL_3$$

because BC & CD are decreasing

$$dL = \frac{P_1 L_1}{AE} - \frac{P_2 L_2}{AE} - \frac{P_3 L_3}{AE}$$

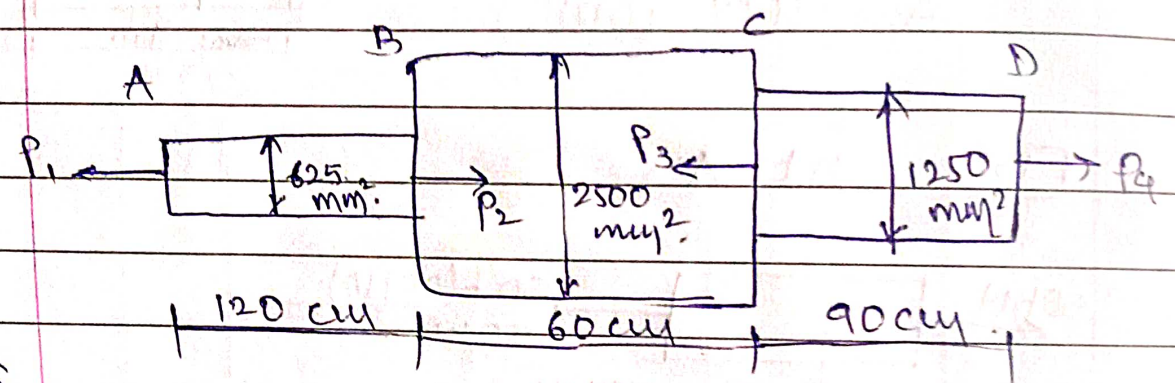
$$= \frac{50(600)}{1000 \times 1 \times 10^5} - \frac{30(1000)}{1000 \times 1 \times 10^5} - \frac{10(1200)}{1000 \times 1 \times 10^5}$$

$$dL = -0.12 \text{ mm}$$

-ve sign indicates decrease in length of bar.

IMP
Q

A member ABCD is subjected to point load P_1, P_2, P_3, P_4 as shown in fig. Calculate the force P_2 if $P_1 = 45 \text{ kN}$, $P_3 = 450$, $P_4 = 130 \text{ kN}$. Determine the total elongation of ~~rod~~^{member}. Assuming modulus of elasticity $E = 2.5 \times 10^5 \text{ N/mm}^2$.



- $P_1 = 45$
- $P_2 = 365$
- $P_3 = 450$
- $P_4 = 130$

$$P_1 + P_3 - P_2 - P_4 = 0$$

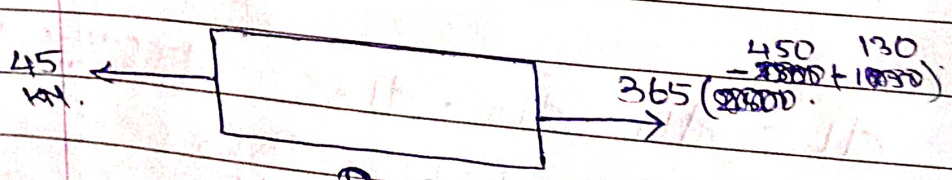
$$P_1 + P_3 = P_2 + P_4$$

$$45 + 450 = P_2 + 130$$

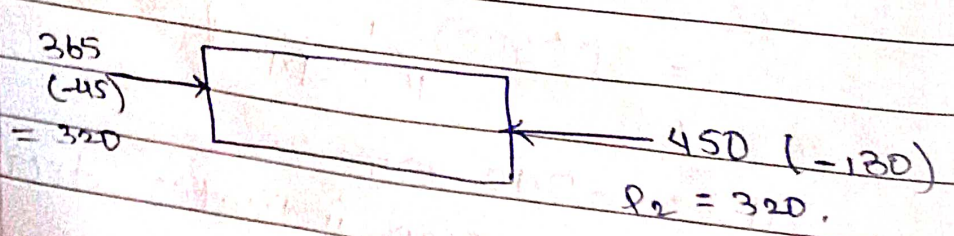
$$P_2 = \underline{\underline{365 \text{ kN}}}$$

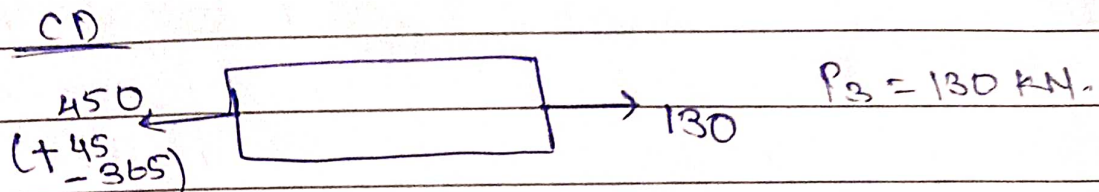
FBD

AB



BC





$$\Delta L = \Delta L_1 - \Delta L_2 + \Delta L_3$$

$$\Delta L_1 = \frac{P_1 L_1}{A_1 E} - \frac{P_2 L_2}{A_2 E} + \frac{P_3 L_3}{A_3 E}$$

$$= \frac{45(1200)}{625(2.5 \times 10^5)} - \frac{320(600)}{2500(2.5 \times 10^5)} + \frac{130(900)}{1250(2.5 \times 10^5)}$$

$$\Delta L = \underline{\underline{0.4918 \text{ mm.}}} \text{ (extension)}$$

~~Prp~~

A tensile load of 40 kN. is acting on a rod of diameter 40 mm & of length 4 m. A bore of diameter 20 mm is made centrally on the rod. To what length the rod should be bored so that the total extension will increase 30%. Under the same tensile load take $E = 2 \times 10^5 \text{ N/mm}^2$.

